Bridging Units: Resource Pocket 5

Set notation, number lines and Venn diagrams

These topics are not listed explicitly in 2007 Key Stage 3 Programme of Study.

Students may be used to the concept of number lines though early work on the development of number skills, for example ‘counting on’ to carry out a subtraction calculation; however, it is possible that number lines were last used in primary school or the early years of secondary school and so it is useful to refresh these concepts. In addition, the use of ‘filled’ and ‘empty’ circles to represent whether or not an inequality is a strict inequality will be new to most students, although inequalities are listed in the current Key Stage 3 Programme of Study.

The formal use of set notation is not included in the current GCSE but is indicated in reference A22 (Higher tier only) where solution sets for inequalities must be represented ‘using set notation’ as well as on a graph and/or number line. In this resource pocket the basic notation involved in using sets will be introduced.

Venn diagrams are listed in the probability section of the new Key Stage 3 Programme of Study and occur in GCSE reference P6 which involves representing sets and combinations of sets using Venn diagrams, as well as through the use of tables and grids. In this resource pocket, the basic use of Venn diagrams will be introduced and links made with set notation.

Some of the activities are quite demanding and are indicated as being more suitable for students who are progressing to study the Higher tier GCSE specification, especially set notation.

This resource pocket progresses through three sections: developing understanding, skills builders and problem solving activities. As with all 9 resource pockets there are a number of different learning styles and approaches used to cater for a variety of learners.

1 Developing Understanding

These are class based, teacher led questions with suggested commentary to get the most from a class or small group discussion. The boxed text can either be copied onto the whiteboard for class discussion, or printed onto cards and handed out to students to be used for paired or small group work.

2 Skills Builders

These are standard progressive worksheets that can be used to drill core skills in a particular area.

3 Problem Solving Activities

Extension activities for paired work or small group work to develop problem solving skills whilst focussing on a particular area of mathematics that students can learn to apply.

Developing Understanding 1

What is the meaning of the statement $3 < x \leqslant 8$?

Which numbers are included in the interval shown?

Which numbers are not included?

Why are two different **inequality** symbols used?

This activity is to encourage students to think about the use of inequality notation to represent regions on a number line. The information in the box should be displayed on the whiteboard, or the inequality could be written on the whiteboard.

Using the information in the box above, lead a class discussion by asking students to write down one or more numbers on their mini-whiteboards that would fall in the interval. Follow this up by asking which numbers are not included in the interval. Draw out 'unusual' examples listed by students, for example decimals or the inclusion of the numbers 8 or (incorrectly) 3.

Some questions that would develop the discussion are:

- What does the word 'inequality' mean?
- If we wanted to include 3, how would we write the inequality?
- How do we say $3 < x \leqslant 8$ in words?
- If we just wanted to say 'x is bigger than 3', how would be write the interval?
- How many numbers lie in the interval $3 < x \leqslant 8$?

The last question is quite difficult - there are an infinite number of values in the interval as we can continually increase the number of decimal places. This is conceptually quite difficult so is best targeted only at more able students.

Now ask students to write down an interval on their mini-whiteboards which includes particular values that you state eg 7; 13.5; $\sqrt{2}$ etc… Choose values that are appropriate to the group. Discuss any unusual intervals listed by the students.

Now display the second box of information (see next page). Encourage students to spend some time in pairs or small groups discussing the diagrams and what the symbols might mean.

Ask students:

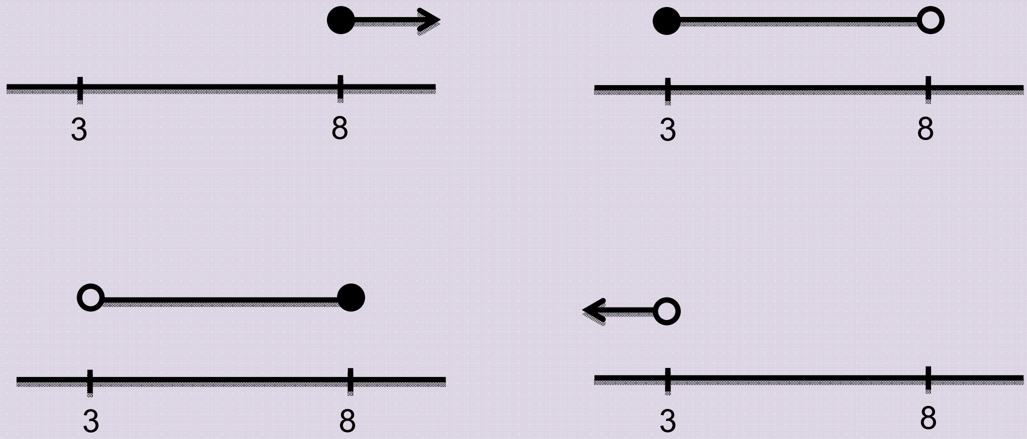
- What inequality is represented by the other three diagrams?
- What might the arrows pointing 'outwards' mean?
- What is indicated by the 'filled' and 'empty' circles?
- Is the scale on the axis important?

For the final question, students need to understand that the axis does not have to be marked to scale - it is just a representation of the interval. The filled circles represent inequalities which

.

The interval $3 < x \leqslant 8$ is going to be shown in a diagram.

Which of these diagrams do you think is correct?

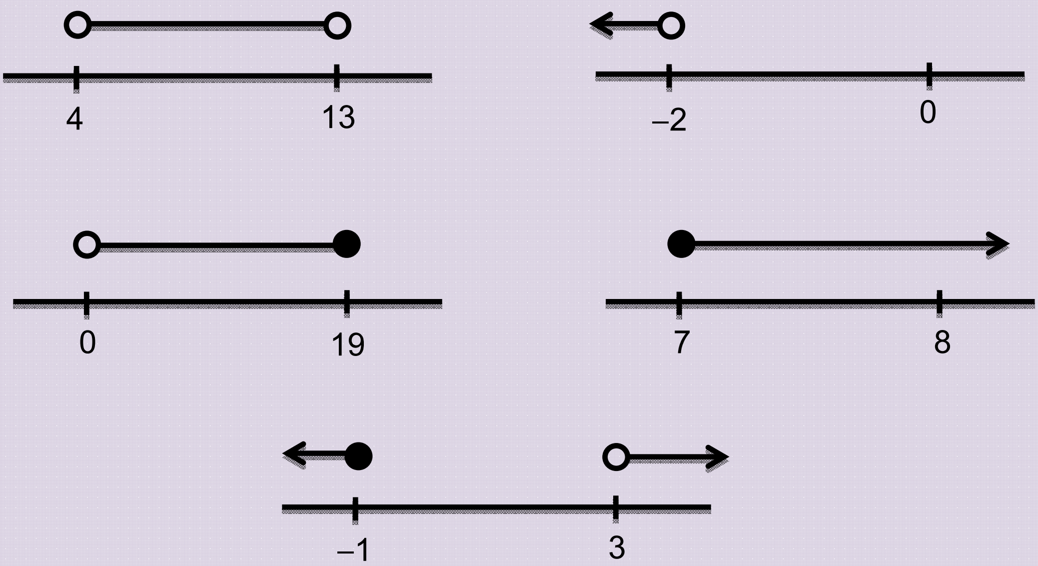


What do you think each of the symbols mean?

What interval is represented by the other three diagrams?

The bottom left hand diagram is the correct answer. The other intervals are: $x \geqslant 8$ (top left); $3 \leqslant x < 8$ (top right); and $x < 3$ (bottom right).

Now display the third box (below) which shows various intervals displayed as diagrams. These could be displayed one by one or as a whole. Ask students individually, or in pairs, to identify the intervals and write them on their mini whiteboards using inequality notation.



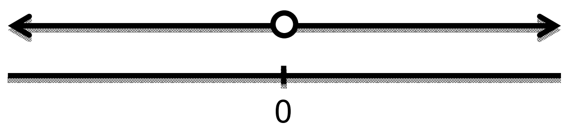
The answers (clockwise from top left) are: $4 < x < 13$; $x < -2$; $x \geqslant 7$; $x \leqslant -1$ and $x > 3$ (two separate intervals); $0 < x \leqslant 19$

Students should note that some of the numbers on the number lines are not required.

Finally, if time allows, show students the fourth box (below) and ask them to draw number lines to represent these intervals. Check student responses carefully and discuss any cases of disagreement to draw out the correct answers.

1	$x \geqslant 5$	**2**	$-1 < x < 5$
3	$x < 3$ and $x \geqslant 9$	**4**	$x < 4.5$
5	$-2 \leqslant x \leqslant 6$	**6**	$x \neq 0$

Question 6 is the only tricky case, and this should only be used with students who are more confident with the previous examples. As a prompt, ask students how they would draw $x > 0$ and $x < 0$ and how these might be combined. The correct answer is:



Students should now feel confident in expressing an interval in inequality notation and using number line diagrams. Aspects of this topic will be considered in Developing Understanding 2, which considers set notation.

Developing Understanding 2

This section introduces the idea of set notation. Simple examples from mathematics and 'real life' will be used to outline the key concepts. For students who will progress to study the Foundation tier at GCSE, completion of the activities in the first two boxes will be sufficient. The notation for intersection, complement and union of sets is introduced to extend more able students. Although this is unlikely to be assessed at GCSE, it is beneficial for students to learn more about mathematical notation, especially if they are thinking of progressing to AS level or A level.

Joe is writing down the set of all the mathematical shapes he can think of.

He calls his set S

What shapes could he include in S

Display the box above on the board. Lead a discussion in to the possible shapes that could be included in set S. Encourage students to think about:

- The number of sides
- Lines of symmetry / rotational symmetry
- 2D or 3D shapes;
- Shapes with special names e.g. trapezium, rhombus
- Correct terminology e.g. prism, pyramid, isosceles

but do not direct students too much.

Write the list of shapes on the board using 'curly bracket' notation,

ie S = {kite, right-angled triangle, square based pyramid, …}

Joe decides to split his set into a number of smaller sets.

He labels his sets as follows

T = the set of 3D shapes

Q = the set of quadrilaterals

P = the set of shapes that start with the letter 'p'

What shapes might be in each of his sets?

Give students a couple of minutes to work in pairs, perhaps on mini whiteboards. Challenge students to think of as many shapes as possible for each set. Hopefully students will identify the fact that some shapes can go in more than one set. Ask students:

- Are there any shapes that would go in both set T and Q?
- Are there any shapes that would go in both set Q and P?
- Are there any shapes that would go in both set T and P?

Introduce the word 'intersection' – shapes that fall in two sets are said to be in the intersection of the two sets. For more able students the formal notation ∩ can be introduced (note that ∩ is **not** a letter n). For the questions above, the answers would be:

- T∩Q is empty. There are no shapes that are both three-dimensional and a quadrilateral, as quadrilaterals are two-dimensional shapes. The 'empty set' notation φ could be introduced.
- Q∩P = {parallelogram}
- T∩P = {pyramid, prism, parallelepiped, pentagonal prism, pentagonal pyramid, polyhedron}

Next, introduce students to the idea of the 'complement' of a set (stress the spelling, and the difference from 'compliment'). Tell students that the complement of a set is denoted by a dash after the set name e.g. T' and that this means 'the shapes that are not in the set'.

Ask students to list possible **elements** of T', Q' and P'. Some possible responses might be:

- T' = {kite, square, triangle, etc…}
- Q' = {pentagon, triangle, decagon, pyramid, etc…}
- P' = {rhombus, arrowhead, decagon, cone, etc…}

Next, display the following box on the board. Ask students to work in pairs or small groups to list the elements of the sets stated.

Reshma is sorting the numbers 1 to 30 into sets.

Her sets are:

E = the set of even numbers

P = the set of prime numbers

S = the set of square numbers

C = the set of cube numbers

Which numbers are in each of her sets?

The answers are:

- E = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26,28, 30}
- P = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}
- S = {1, 4, 9, 16, 25}
- C = {1, 8, 27}

Introduce the idea of a union of two sets, using the notation ∪. This symbol means that then two sets are joined together to make a new, larger set. So, for example,

S∪C = {1, 4, 8, 9, 16, 25, 27}

Note that 1 is in both sets but does not need to be listed twice in the union.

To finish this section, either read out the expressions in the box below, or display the box and ask students to list the elements of the sets.

1	E∩P	**2**	P'	**3**	S∪P	**4**	P∩S
5	(E∪C)'	**6**	E∩S	**7**	(E∩S)'	**8**	S∩C

The answers are:

1 E∩P = {2} Note that 2 is the only even prime number

2 P' = {1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28, 30}

3 S∪P = {1, 2, 3, 4, 5, 7, 9, 11, 13, 16, 17, 19, 23, 25, 29}

4 P∩S = ∅ Note that there are no prime numbers that are also square numbers

5 (E∪C)' = {3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 29}

6 E∩S = {4, 16}

7 (E∩S)' = {1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30}

8 S∩C = ∅ Note there are no square numbers under 30 that are also cube numbers.

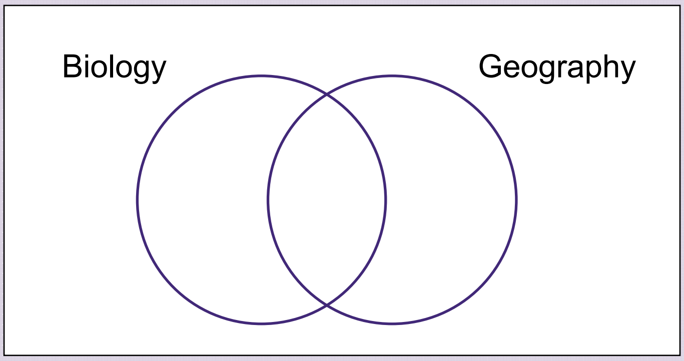
Developing Understanding 3

This section introduces Venn diagrams, which are not on the current Key Stage 3 Programme of Study but are listed in the new Key Stage 3 Programme of Study and in the new Foundation tier GCSE.

Display the diagram below on the board and ask students to think about what each section of the diagram might represent. Include the region outside the two sets, which represents people who study neither Biology nor Geography.

This is a good time to introduce the term 'universal set', which is the set containing all of the elements being considered – in this case the students in Year 12 at Newtown College.

A diagram is going to be drawn to represent the subject choices of the Year 12 students at Newtown College



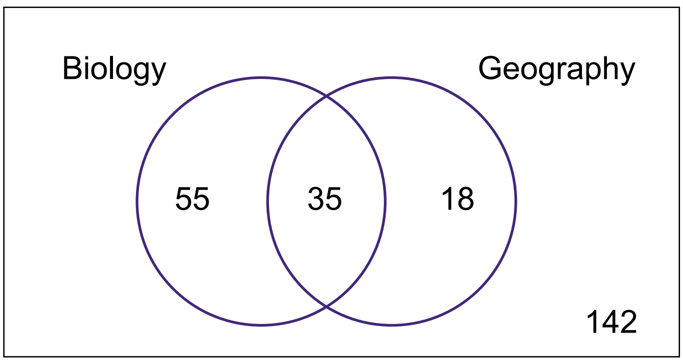
What do you think each section of the diagram represents?

Now provide students with the following information and ask them to draw a completed Venn diagram on their mini whiteboards – there should be one number in each section of the diagram.

- 90 students take Biology;
- 35 students take both Biology and Geography;
- 18 students take Geography but do not take Biology;
- There are 250 students in total in Year 12.

Ask students to explain their reasoning. Draw out the importance of subtracting the number of students who do both subjects from the number of students who do Biology to obtain the number of students who **only** do Biology (but not Geography). Similar calculations are needed to work out the number of students who do neither subject.

The solution is



If students have completed *Developing Understanding 2* on set notation, ask them what the following set notation would represent in this situation:

- B∩G
- G'

The answers are: ‘Students who take Biology and Geography’ and ‘Students who do not take Geography’.

Now ask students how we could write the following in set notation:

- Students who do not take Biology
- Students who take Biology but do not take Geography
- Students who take neither Geography nor Biology

The answers are:

- B'
- B∩G'
- G'∩B' or (G∪B)'

Now say that a student is going to be chosen at random from all of the students in Year 12 at Newtown College. What is the probability:

- They study neither Biology nor Geography?
- They study Geography but not Biology?
- They study Geography?

The answers are: $\frac{142}{250}$, $\frac{18}{250}$ and $\frac{53}{250}$ (or simplified versions of these fractions).

If further consolidation is required, the following scenario could be used.

Ask students to complete a Venn diagram and then devise their own questions to ask a friend. Encourage the use of set notation where possible.

A group of 200 Year 7 students are asked if they own a dog or a cat.

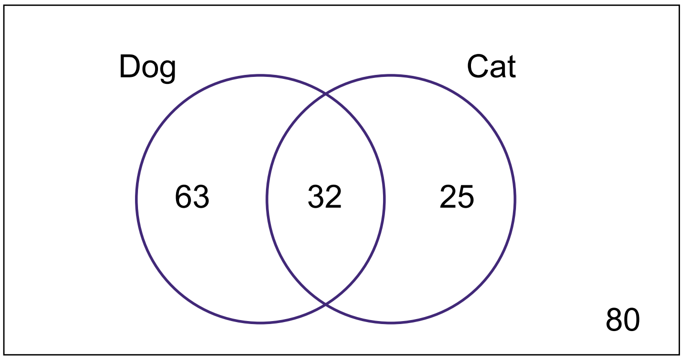
- 32 own both a dog and a cat;
- 25 students only have a cat;
- 95 students own a dog.

Produce a Venn diagram to represent this situation.

Devise a set of questions to ask a friend about the Venn diagram.

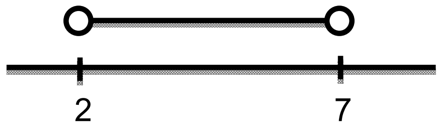
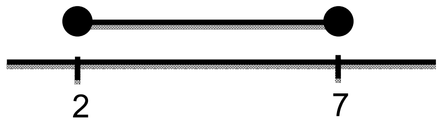
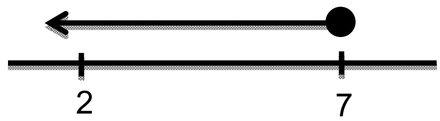
Include some set notation and some probability questions.

The Venn diagram to represent this situation would look like this:



Skills Builder 1: Number lines and Inequalities

Match up the following cards, pairing one number line with one inequality. Two cards in each column do not have a pair - fill in the missing inequalities/number lines to complete these pairs.

$2 \leqslant x \leqslant 7$	1	A	
$x > 7$	2	B	
	3	C	
$2 < x \leqslant 7$	4	D	
$x \leqslant 2$	5	E	
	6	F	
$x > 2$	7	G	
$2 \leqslant x < 7$	8	H	
$x \leqslant 7$	9	I	
$2 < x < 7$	10	J	

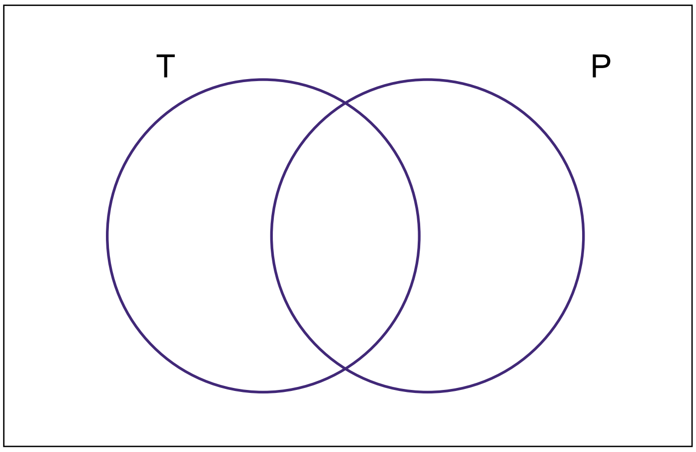
Skills Builder 2: Set notation and Venn diagrams

1 Louis writes down a set of quadrilaterals Q = {rhombus, parallelogram, square, kite, rectangle, arrowhead, isosceles trapezium, trapezium}.

He wants to draw a Venn diagram which split Q into the following sets:

- T = {Quadrilaterals with **at least** one line of symmetry}
- P = {Quadrilaterals with **at least** one pair of parallel sides}

Complete the Venn diagram below, writing each quadrilateral in the correct section.



2 The set S contains the numbers 1 to 20. Some sets within S are:

E = {even numbers}

F = {factors of 24}

M = {multiples of 3}

(a) List the elements of each of the sets E, F and M.

(b) List the elements of the following sets:

(i) $E \cap M$

(ii) F'

(iii) $E \cup M$

(iv) $F \cap E$

(v) $(F \cup M)'$

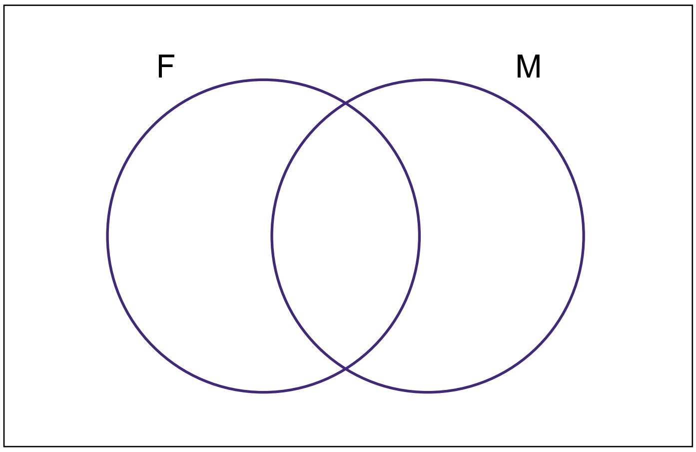
(vi) $F \cap M$

Skills Builder 3: Venn diagrams and probability

1 A group of 50 children are asked if they like drinking fruit juice (F) or milk (M) for their school lunch.

13 students said they like both drinks; 11 only like milk and 20 children like drinking fruit juice.

Complete the Venn diagram below.



What is the probability that a student chosen at random:

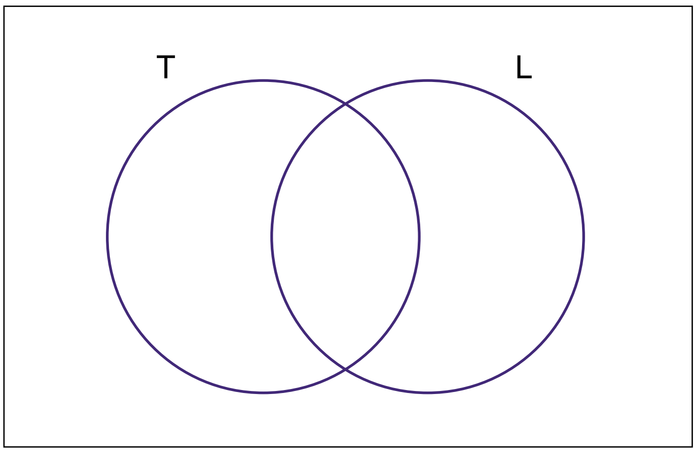
(a) does not like drinking fruit juice or milk?

(b) likes only one of the drinks?

(c) likes milk?

2 A police officer keeps records of the faults on 90 cars he checks. 62 cars have no faults, 17 cars have an illegal tyre (T) and 20 cars have a light that does not work (L).

Complete the Venn diagram below.



What is the probability that a car chosen at random from the 90 cars:

(a) has both faults?

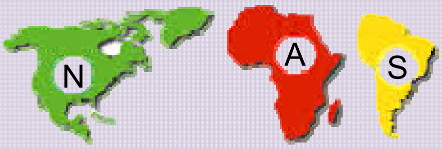
(b) has a light that does not work, but has legal tyres?

(c) has at least one fault?

Problem solving 1: Harder Venn diagrams (3 sets)

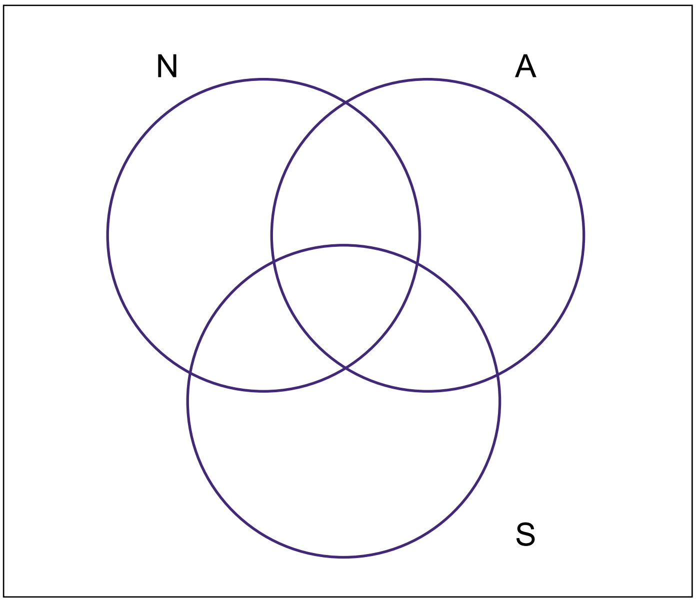
Use the following information to complete the Venn diagram shown.

A travel company asks 200 University students are asked if they have ever visited the continents of North America (N), Africa (A) or South America (S).



The results were as follows:

- 9 had visited all three continents;
- 18 had visited North America and Africa;
- 23 had visited North America and South America;
- 102 had visited none of the three continents;
- 13 had visited only Africa;
- 40 had visited exactly two continents;
- Twice as many students had visited only North America than had only visited South America.



If a student is picked at random from the sample, what is the probability that they:

(a) had visited exactly one of the three continents?

(b) had visited North or South America but not both?

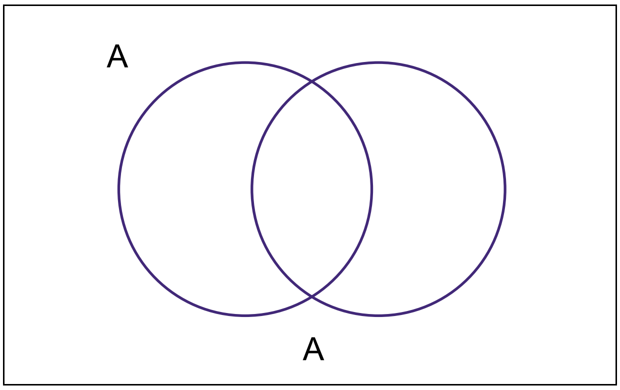
(c) had visited at most one of the three continents?

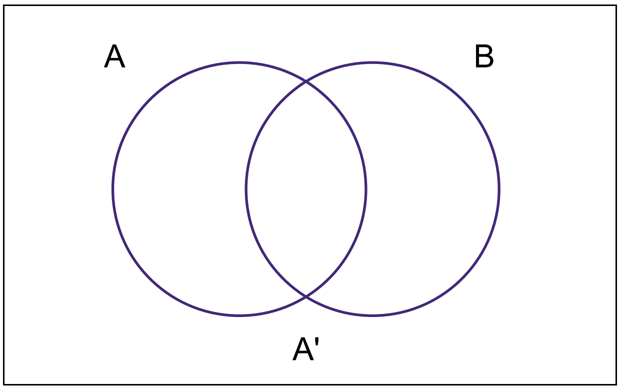
If there are 6000 students at the university, how many would be expected to have visited two or more of the three continents?

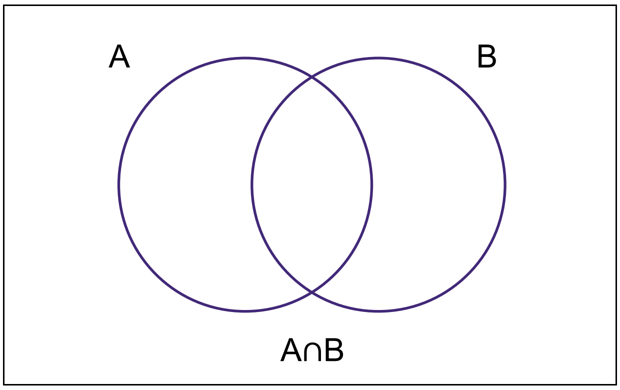
Problem solving 2: Harder sets (shading regions)

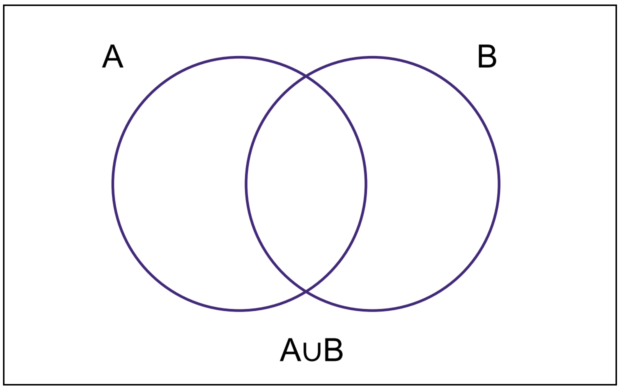
This activity would be more suitable for students progressing to GCSE Higher tier, but is also accessible to Foundation tier students.

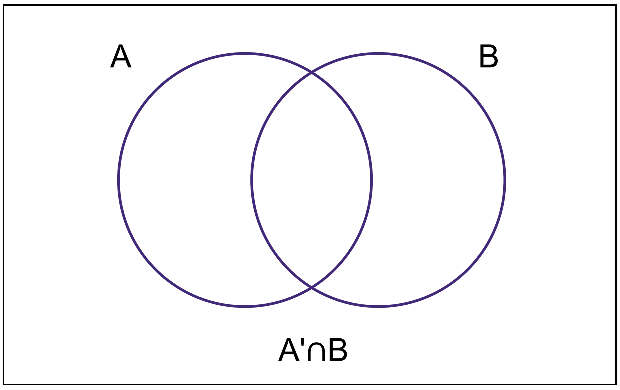
For each region stated, shade the appropriate region on the Venn diagram.

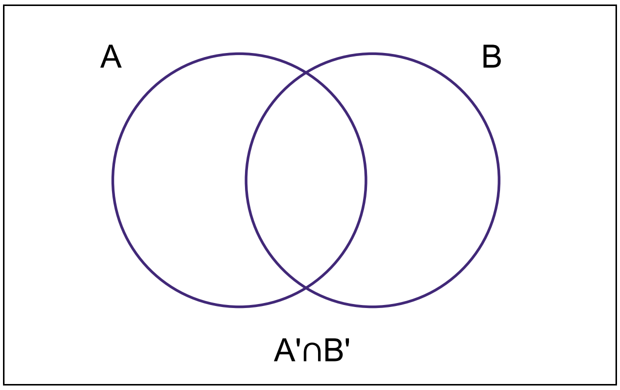


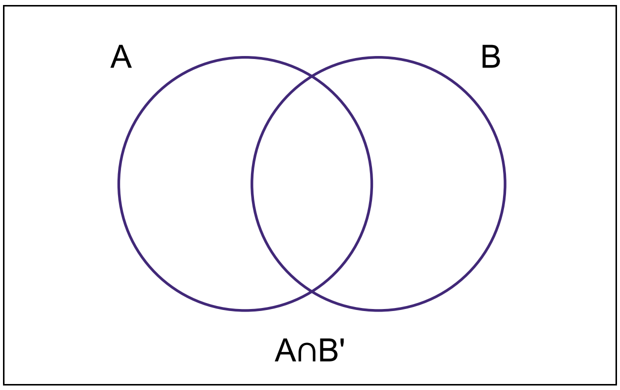


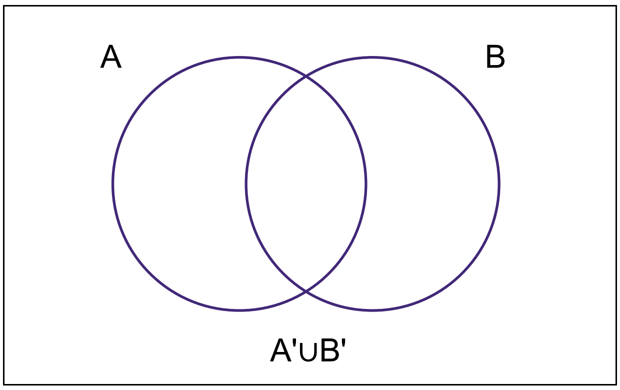












Answers

Skills builder 1: Number lines and inequalities

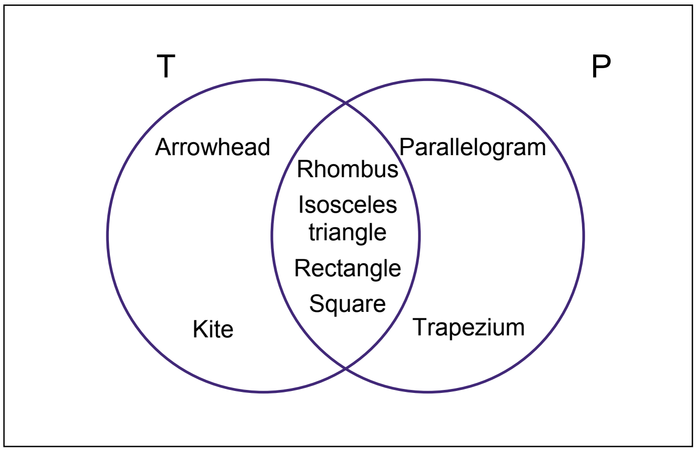
The pairings are: 1D, 2J, 4I, 7G, 8C, 9F, 10B.

Cards 3 and 6 do not have inequalities – one of these matches to A; the other matches to a blank card in the right hand column (either E or H) so that students can create a complete new pair.

The other blank picture card (E or H) pairs with card 5.

Skills builder 2: Set notation and Venn diagrams

1



2

(a) E = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20} F = {1, 2, 3, 4, 6, 8,12} M = {3, 6, 9, 12, 15, 18}

(b) (i) E∩M = {6, 12, 18}

(ii) F' = {5, 7, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20}

(iii) E∪M = {2, 3, 4, 6, 8, 9, 10, 12, 14, 15,16,18, 20}

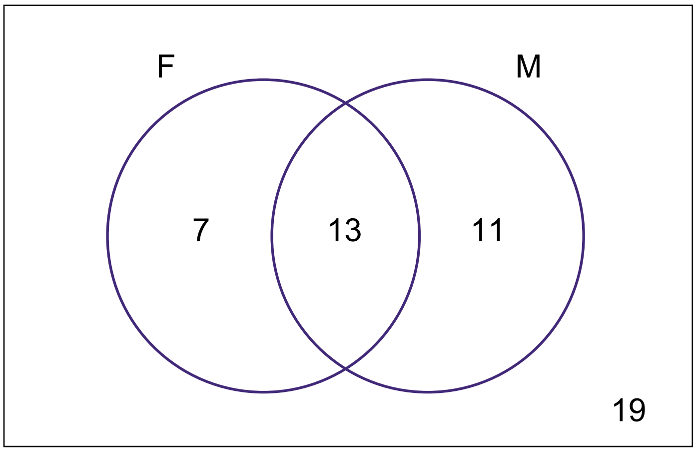
(iv) F∩E = {2, 4, 6, 8, 12}

(v) (F∪M)' = {5, 7,10, 11, 13, 14, 16, 17, 19, 20}

(vi) F∩M = {3, 6,12}

Skills builder 3: Venn diagrams and probability

1



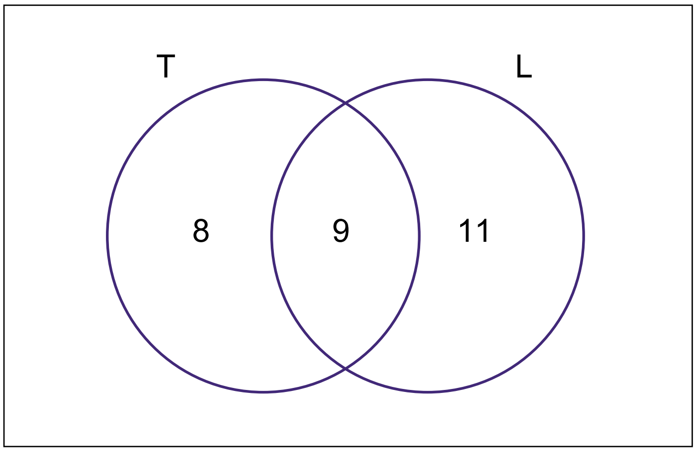
What is the probability that a student chosen at random:

(a) p (does not like drinking fruit juice or milk) = $\frac{19}{50}$

(b) p (likes only one of the drinks) = $\frac{18}{50}$

(c) p (likes milk) = $\frac{24}{50}$

2



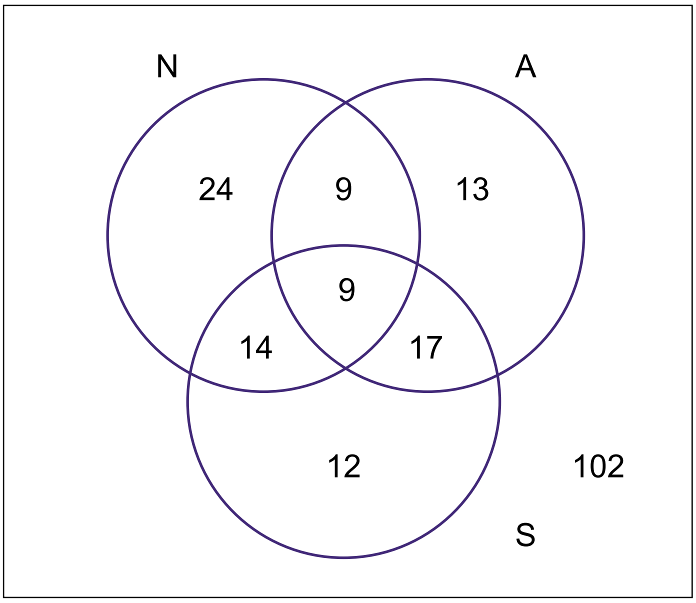
As 62 cars have no faults, we know the total of the three sections inside the Venn diagrams must be 90 – 62 = 28 ie 28 cars have one or more faults. We know 17 have illegal tyres and 20 have a faulty light - this totals 37 and so 9 cars (37 – 28) must have both faults. This allows the rest of the Venn diagram to be completed.

(d) p (has both faults) = $\frac{9}{90}$

(e) p (has a light that does not work, but has legal tyres) = $\frac{11}{90}$

(f) p (has at least one fault) = $\frac{28}{90}$

Problem solving 1 : Harder Venn diagrams (3 sets)



To complete a Venn diagram with three circles, students should work from the centre outwards. For example, as 9 students have visited all three continents, 9 must be subtracted from 18 to identify the number of students who had only visited North America and Africa (but not South America). Students should think about which of the regions which represent 0 continents, 1 continent etc…

(a) p (had visited exactly one of the three continents) = $\frac{49}{200}$

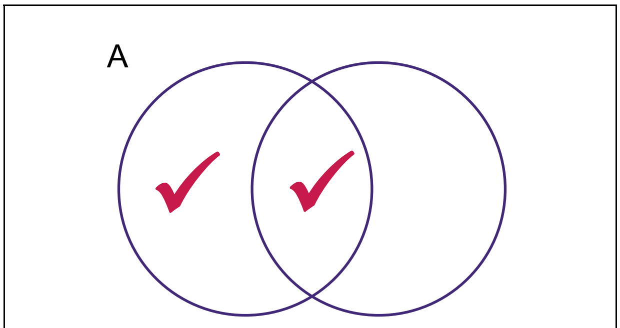
(b) p (had visited North or South America but not both) = $\frac{36}{200}$

(c) p (had visited at most one of the three continents) = $\frac{151}{200}$

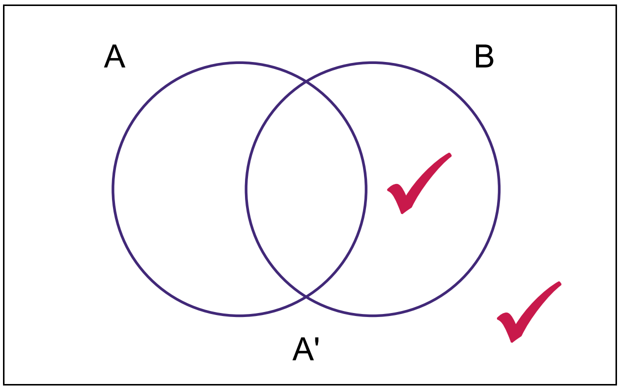
49 out of 200 students have visited two or more of the three continents, so for 6000 students we would expect $\frac{49}{200} \times 6000 = 1470$ to have attended two or more continents.

Problem solving 2 : Harder sets (shading regions)

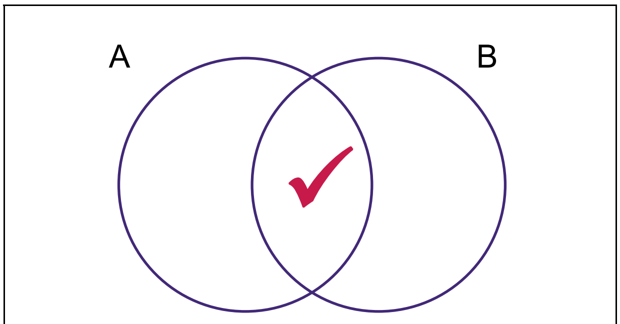
Ticks indicate regions that should be shaded.



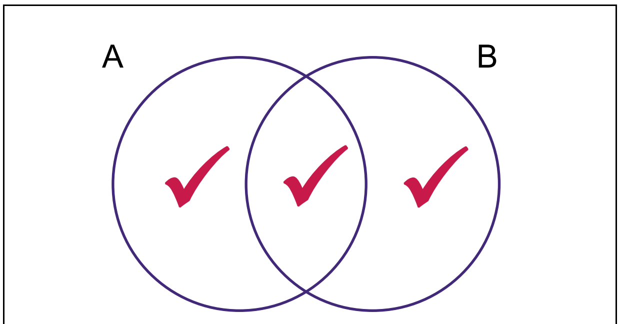
A



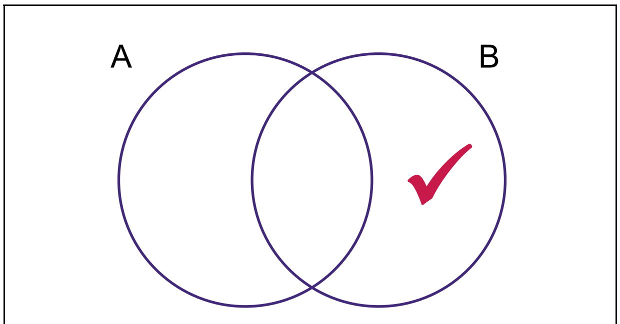
A'



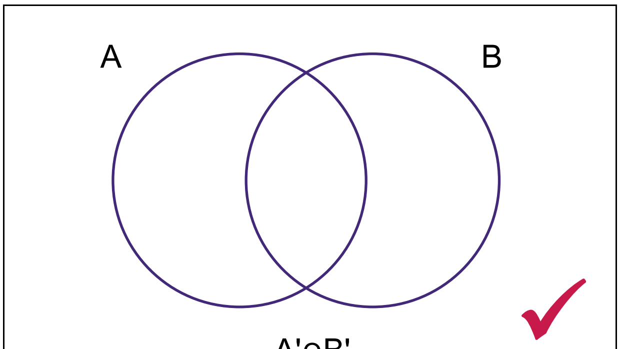
A∩B



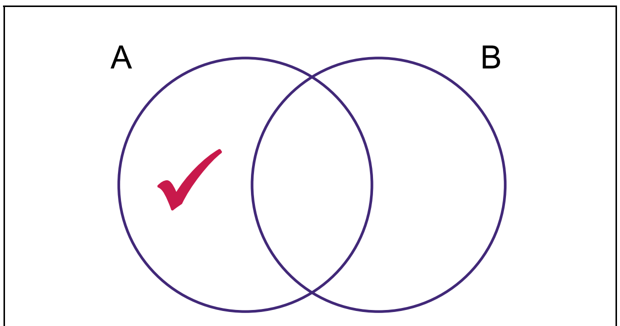
A∪B



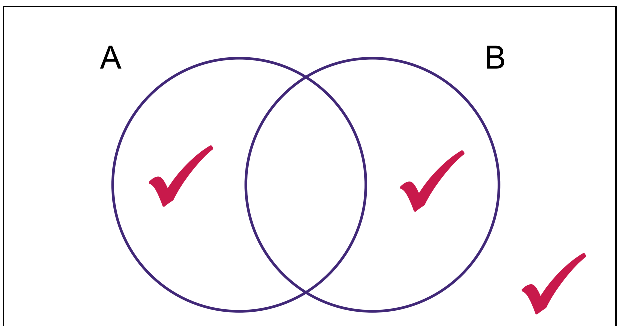
A'∩B



A'∩B'



A∩B'



A'∪B'